Task 2.

1.Exact solution:

y’ = -2\*y +4\*x y(0)=0

y’+2\*y=4\*x

y=u\*y1

y1’+2\*y1=0

y1’/y1=-2

ln|y1|=-2\*x

y1=e^(-2\*x)

y = u\*e^(-2\*x)

u’\*e^(-2\*x)=4\*x

u’=(4\*x\*e^(2\*x))’

u=2\*x\*e^(2\*x) - e^(2\*x) +c

y = (2\*x\*e^(2\*x) - e^(2\*x) +c)\*e^(-2\*x)

y = 2\*x+c\*e^(-2\*x)-1

y(0)=c\*e^(0)-1=0

c=1

y = 2\*x+e^(-2\*x)-1

2.Structure of the program:

In my program I have functions: euler, impr\_euler,runge\_kutta where I provide numerical solutions for the given function. They refer to f, the original task. I use function real\_f to find the exact solution. I also havefunctions show\_exact, show\_euler, show\_imp\_euler, show\_runge, show\_all\_methods to show different graphs for different methods if we need to see them separately or at the same time. And also show\_errors and error\_steps for showing errors.

3. Description of each method:

* **def** euler(x,y,h):

**for** i **in** range(1,len(x)):

y[i]=y[i-1]+ h\*f(x[i-1],y[i-1])

This function uses The Euler method to solve f, the original task. For every x from 0 to X we find y, using this formula:



* **def** impr\_euler(x,y,h):

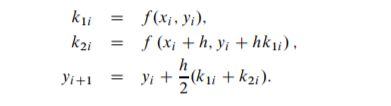
**for** i **in** range(1,len(x)):

k1=f(x[i-1],y[i-1])

k2=f(x[i-1]+h,y[i-1]+h\*k1)

y[i]=y[i-1]+ h/2\*(k1+k2)

This function uses The Improved Euler method to solve f, the original task. For every x from 0 to X we find y, using this formula:



* **def** runge\_kutta(x,y,h):

**for** i **in** range(1,len(x)):

k1=f(x[i-1],y[i-1])

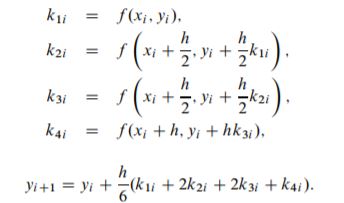
k2=f(x[i-1]+h/2,y[i-1]+(h/2)\*k1)

k3=f(x[i-1]+h/2,y[i-1]+(h/2)\*k2)

k4=f(x[i-1] +h,y[i-1]+h\*k3)

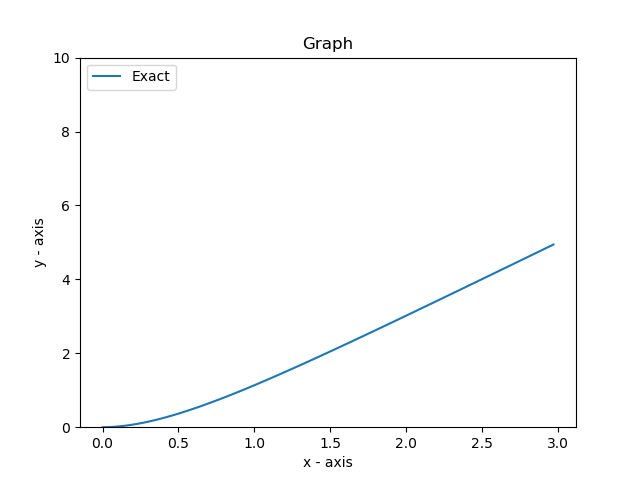
y[i]=y[i-1]+(h/6)\*(k1+2\*k2+2\*k3+k4)

This function uses The Runge-Kutta method to solve f, the original task. For every x from 0 to X we find y, using this formula:

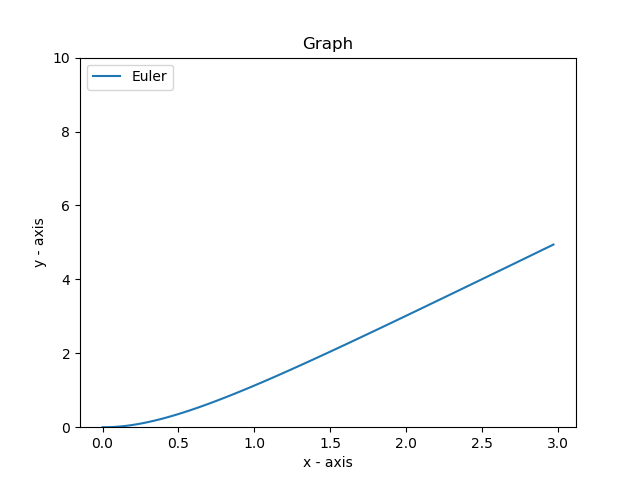


4. Graphs

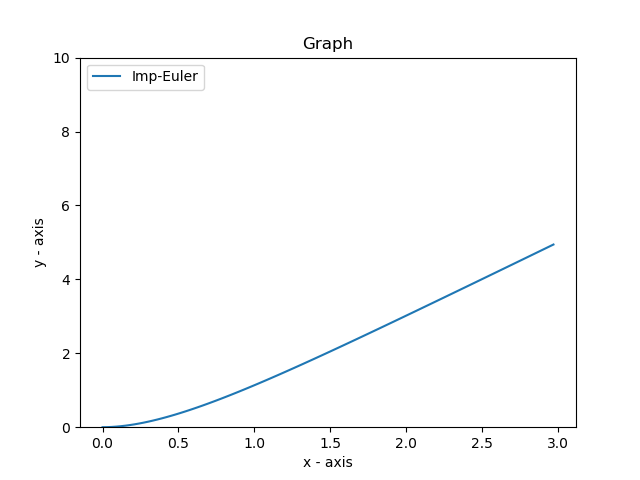
Graph for the exact solution (how other solutions are supposed to look like):



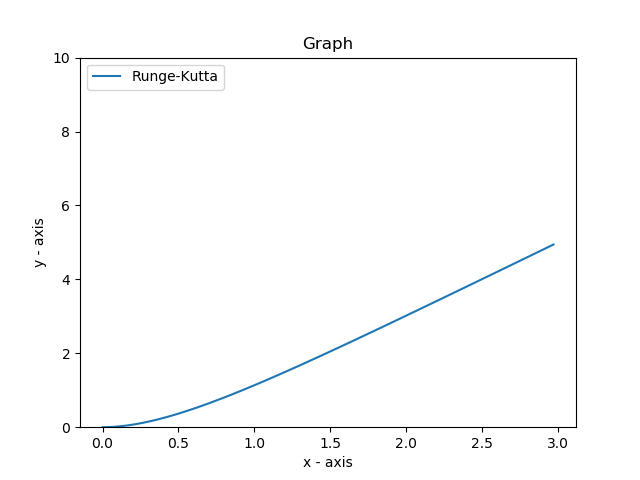
Graph for the Euler method:



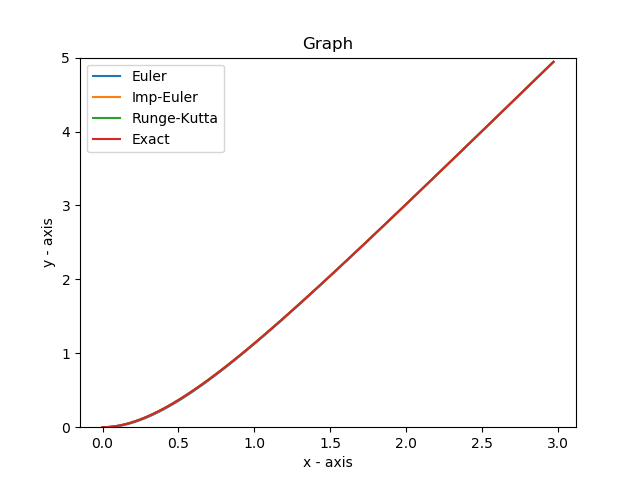
Graph for the Improved Euler method:



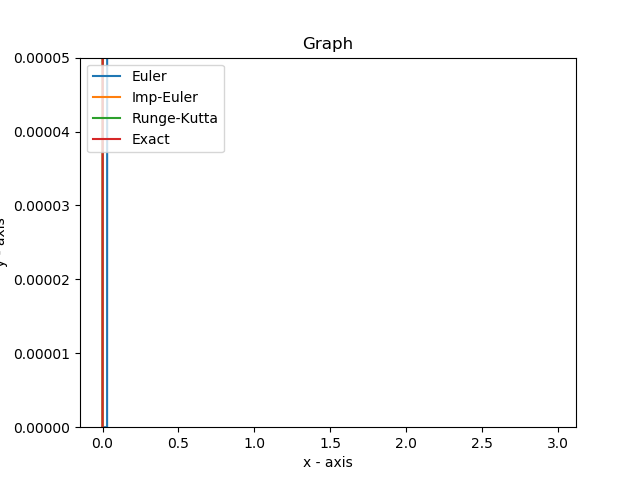
Graph for the The Runge-Kutta method:



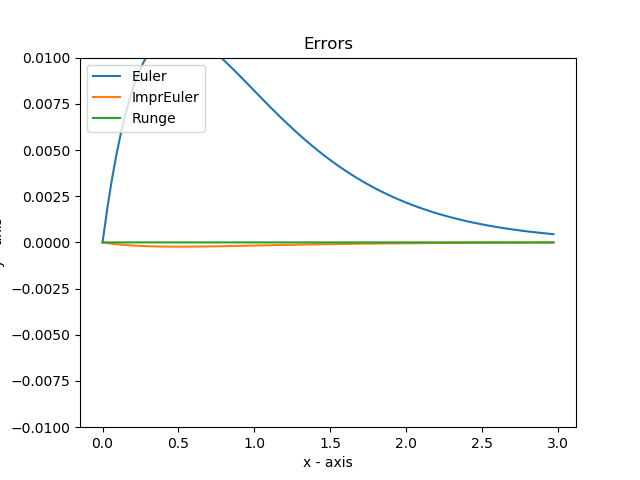
Since they look similar, I would like to show how they look when i put them next to each other:



But the difference between graphs is more obvious if I change limits



And the Error graph



5. Global error.

For finding the best method, we can take a look at the global error:

